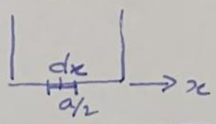
1.1 $[\phi_n(x,t)] = L^{-1/2}$ ($m^{-1/2}$ dans le SI) 0.5

1.2 $\phi_n(x,t)$ amplitude de probabilité de présence
0.5

2.1 $\forall n$ $dP_n = |\phi_n(x,t)|^2 dx$ en $x = a/2$



$$dP_n(a/2) = |\phi_n(a/2)|^2 dx = \frac{2}{a}\sin^2\left(\frac{n\pi}{a}\cdot\frac{a}{2}\right)dx = \boxed{\frac{2}{a}\sin^2\left(\frac{n\pi}{2}\right)dx}$$

2.2

$$\boxed{\rho_n(a/2) = \frac{dP_n(a/2)}{dx}} = \boxed{\frac{2}{a}\times\begin{cases}0 & \text{si } n \text{ est pair}\\ 1 & \text{si } n \text{ est impair}\end{cases}}$$ 1 $\quad dP_n(a/2) = \rho_n(a/2)\,dx$. 1

3. $$\boxed{E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2}{2m}\left(\frac{n^2\pi^2}{a^2}\right)}$$ 1

4. $$E_1 = \frac{\hbar^2 \pi^2}{2ma^2} = \frac{(1.05.10^{-34})^2\pi^2}{2\times 9,1.10^{-31}\times(10^{-8})^2}\cdot\frac{1}{1.6.10^{-19}} = \boxed{37,4\,meV}$$ 0.5

$$\boxed{E_4 = 4\times E_1 = 149,5\,meV}$$ 0.5

Exo1 1. $\boxed{\hat{L}^2 \to \hbar^2 l(l+1) \quad l \text{ entier positif} \quad 0.5}$; $\boxed{\hat{L}_z \to m\hbar \quad -l \leq m \leq l \quad 0.5}$

2. $\hat{L}^2|\psi_+\rangle = \hat{L}^2\times\frac{1}{\sqrt{5}}\left[|1,1\rangle + i\sqrt{4}\,|1,-1\rangle\right] = \hbar^2\times 1(1+1)|\psi_+\rangle \to |\psi_+\rangle$ état propre. 0.5

$\hat{L}^2|\psi_-\rangle = \hat{L}^2\times\frac{1}{\sqrt{5}}\left[\sqrt{4}\,|1,1\rangle - i|1,-1\rangle\right] = \hbar^2\times 1(1+1)|\psi_-\rangle \to |\psi_-\rangle$ état propre 0.5

3. $\hat{L}_z|\psi_+\rangle = \frac{1}{\sqrt{5}}\left[\hbar|1,1\rangle + i\sqrt{4}(-\hbar)|1,-1\rangle\right] = \hbar\frac{1}{\sqrt{5}}\left[|1,1\rangle - i\sqrt{4}|1,-1\rangle\right] \neq \hat{L}_z|\psi_+\rangle$

n'est pas état propre 0.5

$\hat{L}_z|\psi_-\rangle = \frac{1}{\sqrt{5}}\left[\sqrt{4}\hbar|1,1\rangle - i(-\hbar)|1,-1\rangle\right] = \hbar\frac{1}{\sqrt{5}}\left[\sqrt{4}|1,1\rangle + i|1,-1\rangle\right] \neq \hat{L}_z|\psi_-\rangle$

n'est pas état propre 0.5

4. $\hat{L}^2|\psi_\pm\rangle = 2\hbar^2|\psi_\pm\rangle$: résultat $2\hbar^2$ avec une probabilité $= 1$
1 1 1 1

5. $\hat{L}_z|\psi_+\rangle$ résultat $+\hbar$ ou $-\hbar$ avec probabilités 1/5 et 4/5, respectivement

$\hat{L}_z|\psi_-\rangle$ résultat $+\hbar$ ou $-\hbar$ avec probabilités 4/5 et 1/5, respectivement
1 1

6. $$\boxed{\langle\hat{L}_z\rangle_{|\psi_+\rangle} = \frac{\hbar}{5} - \hbar\cdot\frac{4}{5} = -\frac{3}{5}\hbar} \quad 0.5$$ $$\langle\hat{L}_z^2\rangle_{|\psi_+\rangle} = \frac{1}{5}\hbar^2 + \hbar^2\frac{4}{5} = \hbar^2 \to \boxed{\delta\hat{L}_z = \sqrt{\hbar^2 - \frac{9}{25}\hbar^2} = \frac{4}{5}\hbar} \quad 0.5$$

$$\boxed{\langle\hat{L}_z\rangle_{|\psi_-\rangle} = \hbar\cdot\frac{4}{5} - \hbar\cdot\frac{1}{5} = \frac{3}{5}\hbar} \quad 0.5$$ $$\langle\hat{L}_z^2\rangle_{|\psi_-\rangle} = \frac{4}{5}\hbar^2 + \frac{1}{5}\hbar^2 = \hbar^2 \quad \boxed{\delta\hat{L}_z = \sqrt{\hbar^2 - \frac{9}{25}\hbar^2} = \frac{4}{5}\hbar} \quad 0.5$$

7.1 $\langle \psi_+ | \psi_+ \rangle = \frac{1}{5} + \frac{4}{5} = 1$ 0.25 $\langle \psi_- | \psi_- \rangle = \frac{4}{5} + \frac{1}{5} = 1$ 0.25

$$\langle \psi_+ | \psi_- \rangle = \frac{1}{\sqrt{5}} \left[\langle 1,1| - i \sqrt{4} \langle 1,-1| \right] \frac{1}{\sqrt{5}} \left[\sqrt{4} |1,1\rangle + i |1,-1\rangle \right] = \sqrt{\frac{4}{5}} + i^2 \sqrt{\frac{4}{5}} = 0$$

$\langle \psi_+ | \psi_- \rangle = 0$ 0.25 et $\langle \psi_- | \psi_+ \rangle = 0$ 0.25

Ex 02

1. $E = E_c + V(x) =$ constante.
$x<0 \quad E = E_c$; $x>0 \quad E_c = E - V(x) < E$ l'électron ralenti. 1

2. Etat stationnaire $\psi(x,t) = \varphi(x) e^{-iEt/\hbar}$.
$\hat{H}\psi(x,t) = i\hbar \frac{\partial}{\partial t}\psi(x,t) \longrightarrow \hat{H}\varphi(x) = E\varphi(x)$ 0.5

3. $x<0 \quad V(x)=0 \rightarrow -\frac{\hbar^2}{2m}\varphi_I''(x) = E\varphi_I(x) \rightarrow \varphi_I''(x) + \frac{2mE}{\hbar^2}\varphi_I(x) = 0$ 0.25
$x>0 \quad V(x)=V_0 \quad -\frac{\hbar^2}{2m}\varphi_I''(x) + V_0\varphi_I(x) = E\varphi_I(x) \rightarrow \varphi_{II}''(x) + \frac{2m}{\hbar^2}(E-V_0)\varphi_{II}(x) = 0$ 0.25

4. $x<0$ solutions $\varphi_I(x) = A_I e^{ikx} + B_I e^{-ikx}$; $k = \sqrt{\frac{2mE}{\hbar^2}}$ 0.25
$x>0$ solutions $\varphi_{II}(x) = A_{II} e^{iKx} + B_{II} e^{-iKx}$; $K = \sqrt{\frac{2m}{\hbar^2}(E-V_0)}$ 0.25
$B_{II} = 0$ 0.5

5. $\varphi_I(0) = \varphi_{II}(0)$ 0.5 et $\varphi'_I(0) = \varphi'_{II}(0)$ 0.5

6. $A_I + B_I = A_{II}$ 0.25 $k(A_I - B_I) = K A_{II}$ 0.25

7. $r = \frac{B_I}{A_I}$ 0.5
$(K-k)A_I + B_I(K+k) = 0 \rightarrow r = \frac{k-K}{k+K}$ 0.5 >0

8. $R = \frac{|B_I|^2}{|A_I|^2} = |r|^2 = \frac{(k-K)^2}{(k+K)^2}$

9. lui $E \gg V_0$ $k \simeq K$ et $R = 0$ pas de reflexion tout est transmis l'électron ne voit pas la barrière.